Test Name: homework1(Test)

1. It costs a toy retailer $10 to purchase a certain doll. He estimates that, if he charges x dollars per doll, he can sell $80 - 2x$ dolls per week. Find a function for his weekly profit.

f(x) = -10(80-2x) + x(80-2x)

2. Given the following function:

$$f(x) = 8x^3 + 7x^2 - 5$$

Step 1. Find $f(3)$.

$$f(3) = 274$$

Step 2. Find $f(-2)$.

$$f(-2) = -97$$

Step 3. Find $f(x + c)$.

$$f(x) = 8(x+c)^3 + 7(x+c)^2 - 5$$

3. Use the graph to find the indicated limits. If there is no limit, state "Does not exist".

Step 1. Find
$lim_{x \to 1^-} f(x)$. = 2

Answer: $\boxed{}$

A) Does Not Exist

Step 2. Find
$lim_{x \to 1^+} f(x)$. = -5

Step 3. Find
$lim_{x \to 1} f(x)$. = A) Does Not Exist

4. Find the derivative for the following function.

$$f\ (x) = -2x^3$$
$$f'(x) = -6x^2$$

5. Find the derivative for the following function.

$$f\ (x) = \frac{-8}{x^2}$$

$$f'(x) = \frac{16}{x^3}$$

6. Find the derivative for the following function.

$$g(x) = 5\sqrt[3]{x}$$
$$g'(x) = \frac{5}{3\sqrt[3]{x^2}}$$

7. Find the derivative for the following function.

$$y = -2x^{\frac{9}{8}}$$
$$y' = -9\frac{\sqrt[8]{x}}{4}$$

8. Consider the graph of $f\ (x)$. What is the average rate of change of $f\ (x)$ from $x_1 = 0$ to $x_2 = 4$? Please write your answer as an integer or simplified fraction.

Answer: (35-40)/(4-0) = -5/4

9. The cost of producing *x* baskets is given by $C(x) = 630 + 2.4x$. Determine the average cost function.

$$c(x) = \frac{630 + 2.4x}{x}$$

10. Use the Product Rule or Quotient Rule to find the derivative.

$$f\ (x) = (-2x^{-2} + 1)\,(-5x + 9)$$
$$f'^{(x)} = (4x^{-3})(-5x + 9) + (-5)(-2x^{-2} + 1)$$

11. Use the Product Rule or Quotient Rule to find the derivative.

$$f\ (x) = \frac{5x^{\frac{1}{2}} + 7}{-x^3 + 1}$$

$$f\ '(x) = \frac{\left(\frac{5}{2}x^{-\frac{1}{2}}\right)(-x^3 + 1) - (-3x^2)(5x^{\frac{1}{2}} + 7)}{(-x^3 + 1)^2}$$

12. Find the derivative for the given function. Write your answer using positive and negative exponents and fractional exponents instead of radicals.

$$f\ (x) = (3x^{-3} - 8x + 6)^{\frac{4}{3}}$$

$$f'(x) = 4/3(-9x^{-4} - 8)((3x^{-3} - 8x + 6)^{\frac{4}{3}})$$

13. After a sewage spill, the level of pollution in Sootville is estimated by $f\ (t) = \dfrac{550t^2}{\sqrt{t^2 + 15}}$, where t is the time in days since the spill occurred. How fast is the level changing after 3 days? Round to the nearest whole number.

$f'(t) = \dfrac{550t(t^2+30)}{(t^2+15)^{3/2}}$ $\quad f'(3) = 547$

14. The average home attendance per week at a Class AA baseball park varied according to the formula $N(t) = 1000(6 + 0.1t)^{\frac{1}{2}}$ where t is the number of weeks into the season $(0 \text{ £ } t \text{ £ } 14)$ and N represents the number of people.

Step 1. What was the attendance during the third week into the season? Round your answer to the nearest whole number.
2510

Step 2. Determine $N\ '(5)$ and interpret its meaning. Round your answer to the nearest whole number.
The rate of change in attendance in week 5 is +20 people.

15. Consider the following function:

$$3x^3 + 4y^3 = 77$$

Step 1. Use implicit differentiation to find $\frac{dy}{dx}$.

$$-\frac{3x^2}{4y^2}$$

Step 2. Find the slope of the tangent line at $(3, -1)$.

$$-\frac{27}{4}$$

16. Find the intervals on which the following function is increasing and on which it is decreasing.

$$f(x) = \frac{x+3}{x-8}$$

The function is not increasing, but it decreases from $(-\infty, 8) \cup (8, \infty)$

17. A frozen pizza is placed in the oven at $t = 0$. The function $F(t) = 14 + \frac{367t^2}{t^2 + 100}$ approximates the temperature (in degrees Fahrenheit) of the pizza at time t .

Step 1. Determine the interval for which the temperature is increasing and the interval for which it is decreasing. Please express your answers as open intervals.

The temperature is not decreasing, but it increases from $(0, \infty)$. We find this because of the shape of the graph and the global min of F(0)=14.

Step 2. Over time, what temperature is the pizza approaching?

$$\lim_{t \to +\infty} 14 + \frac{367t^2}{t^2 + 100} = 381$$

18. A study says that the package flow in the East during the month of November follows $f(x) = \frac{x^3}{3340000} - \frac{7x^2}{9475} + \frac{42417727x}{1265860000} + \frac{1}{33}$, where 1 £ x £ 30 is the day of the month and $f(x)$ is in millions of packages. What is the maximum number of packages delivered in November? On which day are the most packages delivered? Round your final answer to the nearest hundredth.

$$\sum_{x=1}^{30} \left(\frac{379\,x^3 - 935\,200\,x^2 + 42\,417\,727\,x}{1\,265\,860\,000} + \frac{1}{33} \right) = \frac{13\,326\,123\,963}{1\,392\,446\,000}$$

The maxium number of packages delivered in November is 9.57 million.
On Nov. 23rd the most packages are delivered (local max).

19. Use the Second Derivative Test to find all local extrema, if the test applies. Otherwise, use the First Derivative Test. Write any local extrema as an ordered pair.

$$f(x) = 7x^2 + 28x - 35$$

(-2,-63)

20. Use the Second Derivative Test to find all local extrema, if the test applies. Otherwise, use the First Derivative Test. Write any local extrema as an ordered pair.

$$f(x) = -6x^3 + 27x^2 + 180x$$

(-2, -204) and (5, 825)

21. A beauty supply store expects to sell 120 flat irons during the next year. It costs \$1.60 to store one flat iron for one year. To reorder, there is a fixed cost of \$6 , plus \$4.50 for each flat iron ordered. In what lot size and how many times per year should an order be placed to minimize inventory costs?

S(x) = 1.6 * (x/2)
R(x) = (4.5x + 6) * (120/x)
IC(x) = 1.6 * (x/2) + (4.5x + 6) * (120/x)   0 < x <= 120
IC'(x) = 0.8-720/x^2   IC'(0) = 30
To minimize inventory costs, each lot size should have 30 flat irons and the order should be placed 4 times a year.
Reference: Brief Calculus for the Business, Social, and Life Sciences by Bill Armstrong, Don Davis.

22. A shipping company must design a closed rectangular shipping crate with a square base. The volume is 18432 ft^3. The material for the top and sides costs \$3 per square foot and the material for the bottom costs \$5 per square foot. Find the dimensions of the crate that will minimize the total cost of material.

18432ft^3 = x^2 * y
C = 5*x^2 + 3x^2 + 4xy   C = 8x^2 + 4*(18432/x)
Xmin = 8 * 3^(2/3)   y = 32*3 ^ (2/3)
The height should be 32 * 3 ^ (2/3) feet, the sides should be 8 * 3 ^(2/3) feet.

23. A farmer wants to build a rectangular pen and then divide it with two interior fences. The total area inside of the pen will be 1056 square yards. The exterior fencing costs \$14.40 per yard and the interior fencing costs \$12.00 per yard. Find the dimensions of the pen that will minimize the cost.

x = 24 yards y = 44 yards

1056 = xy
C = 14.40 * 2y + 14.40 * 2x + 12.00 * 2x
C = 14.40 * 2(1056/x) + 14.40 * 2x + 12.00 * 2x
Xmin = 24

24. It is determined that the value of a piece of machinery declines exponentially. A machine that was purchased 7 years ago for \$67000 is worth \$37000 today. What will be the value of the machine 9 years from now? Round your answer to the nearest cent.
37000 = 67000e^(7k)
k = 1/7(log(37))-log(67))
\$17,244.50 = 9 yrs

25. The demand function for a television is given by $p = D(x) = 23.2 - 0.4x$ dollars. Find the level of production for which the revenue is maximized.

R(x) = x*D(x)
R(x) = 23.2x – 0.4x^2
Global max = 29

26. The amount of goods and services that costs \$400 on January 1, 1995 costs \$426.80 on January 1, 2006. Estimate the cost of the same goods and services on January 1, 2017. Assume the cost is growing exponentially. Round your answer to the nearest cent.
426.8=400e ^ (11k)
k = 0.00589554
\$455.40 = 22 yrs

27. A manufacturer has determined that the marginal profit from the production and sale of x clock radios is approximately $380 - 4x$ dollars per clock radio.

Step 1. Find the profit function if the profit from the production and sale of 38 clock radios is \$1700.

$$P(x) = 380x - 2x^2 - 9852$$

Step 2. What is the profit from the sale of 56 clock radios?
\$5,156

28. Use integration by substitution to solve the integral below.

$$\int \frac{-5(\ln(y))^3}{y} dy$$

$-\frac{5}{4}(\ln(y))^4 + \mathrm{C}$

29. It was discovered that after *t* years a certain population of wild animals will increase at a rate of $P'(t) = 75 - 9t^{\frac{1}{2}}$ animals per year. Find the increase in the population during the first 9 years after the rate was discovered. Round your answer to the nearest whole animal.

$$f(x) = \sum_{n=1}^{9} 75 - 9t^{1/2} = 501$$

30. Find the area of the region bounded by the graphs of the given equations.

$$y = 6x^2,\ y = 6\sqrt{x}$$

Enter your answer below.

$$\int_0^1 (6\sqrt{x} - 6x^2)dx = 2$$

31. Solve the differential equation given below.

$$\frac{dy}{dx} = x^3y$$

$\ln(y) + C = \frac{1}{4}x^4 + \mathrm{C}$ or $\mathrm{y} = \mathrm{e}^{\frac{x^4}{4}}$

32. Use integration by parts to evaluate the definite integral below.

$$\int_{-7}^{2} x\sqrt{x + 7}dx$$

Write your answer as a fraction. -144/5

33. The following can be answered by finding the sum of a finite or infinite geometric sequence. Round the solution to 2 decimal places.
A rubber ball is dropped from a height of 46 meters, and on each bounce it rebounds up 22 % of its previous height.

$$f(x) = 46 + \sum_{n=0}^{30} 92(.22)^n$$

Step 1. How far has it traveled vertically at the moment when it hits the ground for the 20^{th} time?
71.95 meters

Step 2. If we assume it bounces indefinitely, what is the total vertical distance traveled?
71.95 meters

34. Find the Taylor polynomial of degree 5 near $x = 4$ for the following function.

$$y = 3e^{5x-3}$$
$$y(4) = 3e^{17}$$
$$y'(4)(x-4) = 15e^{17}(x-4)$$
$$\frac{y''(4)}{2!}(x-4)^2 = \frac{75}{2}e^{17}(x-4)^2$$
$$\frac{y'''(4)}{3!}(x-4)^3 = \frac{125}{2}e^{17}(x-4)^3$$
$$\frac{y''''(4)}{4!}(x-4)^4 = \frac{625}{8}e^{17}(x-4)^4$$
$$\frac{y'''''(4)}{5!}(x-4)^5 = \frac{625}{8}e^{17}(x-4)^5$$

Taylor polynomial for degree 5 near x = 4 is:

$$3e^{17} + 15e^{17}(x-4) + \frac{75}{2}e^{17}(x-4)^2 + \frac{125}{2}e^{17}(x-4)^3 + \frac{625}{8}e^{17}(x-4)^4 + \frac{625}{8}e^{17}(x-4)^5$$